LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **APRIL 2023**

MT 1500 – ALGEBRA, ANALY. GEO., CALCULUS & TRIGONOMETRY

Date: 06-05-2023 Dept. No. Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

PART – A

Answer ALL questions: (10 × 2 = 20)

1. State Leibnitz's theorem.
2. What is the n^{th} derivative of $y = x^m$, if $m < n$?
3. Find the slope of the curve $r = a(1 - \cos\theta)$ at $\theta = \frac{\pi}{2}$.
4. What is the cartesian formula to find the radius of curvature?
5. What is the condition for an equation $f(x) = 0$ to have atleast one root between a and b?
6. Form the equation, one of whose roots is $\sqrt{3} + \sqrt{5}$.
7. Prove that $\sin h^{-1}x = \log_e(x + \sqrt{x^2 + 1})$.
8. Evaluate $\lim_{x\to 0} \frac{tan2x - 2tanx}{x^3}$.
9. Define conic section.
10. Find the pole of the line $2x = y$ with respect to the parabola $y^2 = 2x$.

PART - B

Answer any FIVE questions: (5 × 8 = 40)

11. If $y = \sin ax + \cos ax$, find $\frac{d^n y}{dx^n}$.
12. Prove that the subtangent to the curve $y = a^x$ is of constant length.
13. Find the radius of the curvature of the curve $x^4 + y^4 = 2$ at the point (1,1).
14. Find the pedal equation of the curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with pole at its center.
15. Solve the equation $x^4+2x^3-5x^2+6x+2=0$ given that $1+\sqrt{-1}$ is a root of it.
16. Prove that $32\sin^4 q \cos^2 q = \cos 6q - 2\cos 4q - \cos 2q + 2$.
17. Find the locus of poles of chords of parabola subtending a right angle at the vertex.
18. Find the condition that the lines $lx + my + n = 0$ and $l'x + my' + n' = 0$ may be conjugate with respect to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

PART – C

Answer any TWO questions: (2 × 20 = 40)

19. (a) If $y = \sin(m \sin^{-1}x)$, Show that $(1 - x^2)y_2 - xy_1 + m^2y = 0$ and

$$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} + (m^2 - n^2)y_n = 0. \quad (13)$$

(b) Find the maximum value of the directional derivative at the point $(1,1,-4)$ of the function $\emptyset = 2x^2 + 3y^2 + 5z^2$. (7)

20. (a) Evaluate the maximum value of $x^2 + y^2 + z^2$ when $x + y + z = a$. (7)

(b) Find the positive root of the equation $x^3 - 2x^2 - 3x - 4 = 0$ correct to two places of decimals using Horner's method. (13)

21. (a) Solve the equation $x^3 - 19x^2 + 114x - 216$ given that the roots are in GP. (6)

(b) Separate $\tan h\,(x + iy)$ into real and imaginary parts. (14)

22. (a) Evaluate $\lim_{x\to 0} \frac{tanx + sinx}{\sin^3 x}$. (4)

(b) Prove that the area of the parallelogram formed by the tangents at the extremities of two conjugate diameters of an ellipse is constant and equal to the product of the axes. (16)

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